

Approximate Capacity Region of the MAC-IC-MAC

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Abstract

An approximate capacity region is established of a class of interfering multiple access channels consisting of two multiple-access channels (MACs), each with an arbitrary number of users, with interference from one of the transmitters of one MAC to the receiver of the other MAC, which we refer to henceforth as the MAC-IC-MAC. It is shown that, for the semi-deterministic MAC-IC-MAC, single-user coding at the non-interfering transmitters in each MAC and superposition coding at the interfering transmitter of each MAC achieves a rate region that is within a quantifiable gap of the capacity region, thereby generalizing the result by Telatar and Tse for the 2-user semi-deterministic interference channel. Next, with an explicit coding scheme, we establish an approximate capacity region that is within a one-bit gap of the capacity region for the Gaussian MAC-IC-MAC, thereby extending the work by Etkin *et al* for the two-user Gaussian interference channel. The symmetric generalized degrees of freedom (GDoF) of the symmetric Gaussian MAC-IC-MAC with more than one user per cell, which is a function of the interference strength (the ratio of INR to SNR at high SNR, both expressed in dB) and the numbers of users in each cell, is V-shaped with flat shoulders. An analysis based on the deterministic method shows that, when interference is sufficiently weak or strong, the non-interfering transmitters are not mere sharers of the degrees of freedom with the interfering transmitters. They instead also utilize power levels that cannot be accessed by interfering transmitters (due to the restriction of superposition coding), thereby improving the symmetric sum GDoF to up to one degree of freedom per cell under a range of SINR exponent levels. Time sharing between interfering and non-interfering transmitters is suboptimal.

Index Terms

Approximate capacity, capacity region, constant gap, interference channel, interfering multiple access channel, MAC-IC-MAC, multiple access channel, generalized degrees of freedom.

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I. INTRODUCTION

Due to the rapid increase of data demands in recent years, wireless co-band communication has drawn significant interest in both theory and practice. Bluetooth and Wi-Fi have both been established on 2.4 GHz band, and more recently 3GPP introduced LTE Licensed Assisted Access (LAA) to offload LTE packets to unlicensed spectrum at 5 GHz, which causes Wi-Fi and LAA to coexist. Such emerging technologies motivate the study of co-band interference between cellular networks in network information theory. In this paper, we obtain an approximate capacity region for a class of mutually interfering two-cell networks in which there is interference from one of the transmitters of the MAC to the receiver of the other MAC. For brevity, we refer to this two-cell network as the MAC-IC-MAC. As shown in Fig. 1, the MAC-IC-MAC captures many practical communication scenarios. Fig. 1a shows an uplink cellular network where users $a0$ and $b0$ are located on the cell edge so that interference paths exist between the two cells as indicated by the dashed arrows, meanwhile users $a1$ and $b1$ do not cause interference to the neighboring cell due to their favorable access location in their own cell. Fig. 1b represents a femto cell network, where similar partial interference exists between the macro cell and femto cell. The study of the capacity region of MAC-IC-MAC could therefore provide an approach to increasing uplink throughput and cell edge spectrum efficiency for co-band networks.

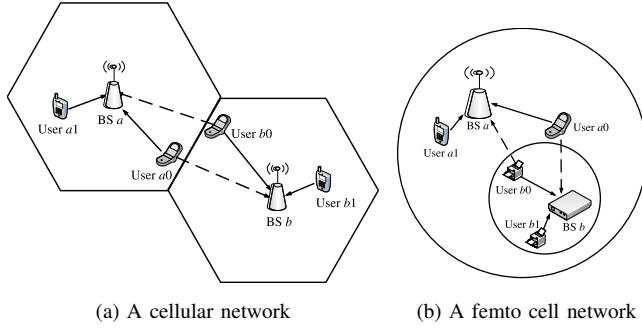


Fig. 1. Examples of the MAC-IC-MAC

The main results of this paper are:

- 1) An achievable rate region for discrete memoryless MAC-IC-MAC: the coding scheme we employ is what would be deemed the most natural for the MAC-IC-MAC: each of the two interfering transmitters employ rate-splitting and superposition coding (as in [1]) and the non-interfering transmitters in each MAC employ single-user random coding. An analysis of such a coding scheme results in an inner bound for the capacity region that is a polytope in a number of dimensions that is more than the total number of transmitters due to rate-splitting at the two interfering transmitters. Such a polytope is defined by an indeterminate number of sum-rate inequalities (indeed, exponentially many in the total number of transmitters) because there is no restriction on the number of users in each cell. Utilizing the particular structure of this polytope however, and using a form of structured Fourier-Motzkin elimination, the two split rates are projected out to obtain a closed-form description of the achievable rate region.

- 2) An outer bound on the capacity region of semi-deterministic MAC-IC-MAC: the proof of this outer bound uses genie-aided decoding arguments and it is shown that it is within a quantifiable gap to the capacity region, thereby extending the corresponding result for the two-user semi-deterministic interference channel obtained by Telatar and Tse [2].
- 3) An inner bound for the Gaussian MAC-IC-MAC that is within a one-bit gap of the capacity region, thereby extending the same result for the Gaussian interference channel due to Etkin *et al* [3].
- 4) The generalized degrees of freedom (GDoF) region for the Gaussian MAC-IC-MAC: the one bit gap capacity for Gaussian MAC-IC-MAC yields its GDoF region. In addition, the symmetric GDoF curve, which is a function of α , the interference strength (the ratio of INR to SNR at high SNR, both expressed in dB) and the number of users in a cell, is V-shaped with flat shoulders on the two sides. The symmetric GDoF curve, which is plotted in Fig. 2, reveals that in a Gaussian MAC-IC-MAC with symmetric cell configuration and equal rate requirement, all transmitters can send information at an approximately interference-free rate in the high SNR regime when $\alpha = \frac{\log \text{INR}}{\log \text{SNR}} \in [0, 1 - \frac{1}{K}] \cup [1 + \frac{1}{K}, \infty)$, where K is the number of users in each cell. As a byproduct, our result also says that the DoF of the K -user symmetric Gaussian MAC-IC-MAC is simply $\frac{1}{K+1}$.

In Section III.E, a few examples will be discussed to demonstrate the achievability of GDoF in various ranges of α using a deterministic analysis. It is shown that the non-interfering transmitters in the MAC-IC-MAC do not always share the DoF with the interfering transmitter in each cell (but when $\alpha = 1$, they do). Using the deterministic approximation, we see that the rate-splitting and superposition coding scheme prevents interfering transmitters to use certain power levels, so that non-interfering transmitters occupy these unused levels, thereby improving the total DoF per cell. Consequently, merely time-sharing between interfering and non-interfering transmitters is suboptimal.

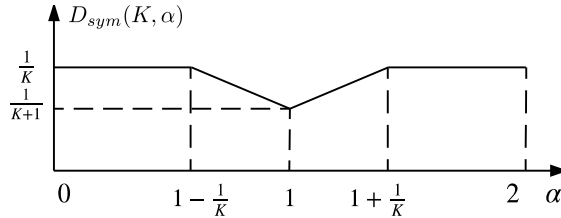


Fig. 2. Symmetric GDoF of Gaussian MAC-IC-MAC

Previous related works are summarized as follows. Multiple access channels are the best understood multi-terminal networks with the capacity region fully determined by Ahlswede [4] and Liao [5]. The key papers on interference channels are [2], [3], [6]–[8]. For the interference channel, the Han-Kobayashi achievable scheme (H-K scheme) in [7] as well as its alternative superposition coding scheme of [1] (referred to henceforth as the CMG scheme) lead to the best inner bound to the capacity region known to date. Telatar and Tse [2] found an outer bound for the so-called semi-deterministic interference channel and quantified the gap to the CMG inner bound. For Gaussian

scalar and vector interference channels, Etkin et al [3] and Karmakar and Varanasi [9] characterized approximate capacity regions to within constant, channel coefficient- and SNR-independent gaps, respectively.

For two cell interfering multiple access channel with an arbitrary number of transmitters in each cell, the work by [10] used interference alignment [11] to achieve interference-free degrees of freedom when the number of users in each cell goes to infinity. Perron et al. [12] defined a type of multiple-access interference channel with only four nodes, where one of the receivers must decode the messages from both transmitters. The capacity region within a quantifiable gap was obtained for the semi-deterministic case. Chaaban and Sezgin studied the fully-connected two-cell multiple access interference channel in which a two-user MAC interferes with a point-to-point link [13]. The capacity region is found for very strong and some cases of strong interference and upper and lower bounds on the sum-rate in the weak interference regime (with the lower bound achievable by treating interference as noise) are also obtained. Subsequently, in [14], they showed that when the interference is weak, treating interference as noise in their model is sub-optimal. The cognitive radio version of that model was studied by the same authors in [15]. Buhler and Wunder [16] derived upper bounds on the sum rate and an achievable scheme for the linear deterministic version of the model in [13]. Fritschek and Wunder obtain a result on the reciprocity between the two-cell deterministic interfering MAC (IMAC) and the two-cell deterministic interfering broadcast channel (IBC) in [17], and obtain an achievable region under a weak interference condition for both those channels. In [18], the deterministic IMAC was revisited using the lower triangular deterministic model introduced by [19], and a constant gap sum capacity as well as the sum GDoF were obtained. For the Gaussian IMAC, Fritschek and Wunder [20] close the gap between the achievable sum rate regions for Gaussian IMAC and the deterministic IMAC. Their coding scheme employs signal scale alignment and lattice coding. Zhu et al. [21] studied the interference Z MAC, a special case of the channel in [13], where they obtained an achievable rate region based on superposition encoding and joint decoding. In the conference version of this paper [22], an approximate capacity region was presented for a special case of the MAC-IC-MAC with a two-user MAC in the first cell and a point-to-point link in the second cell¹

The notation used throughout the paper are summarized as follows. We use capital letters to denote random variables or sequences, such as X and X^n , where $X^n = \{X_1, \dots, X_n\}$. The underlying sample spaces are denoted by \mathcal{X} and \mathcal{X}^n , and specific values of X and X^n by x and x^n . The t -th element of the random vector X_{ij}^n is denoted by $X_{ij,t}^n$. Unless specified explicitly, we will use the usual short hand notation for (conditional) probability distributions where the lower case arguments also denote the random variables whose (conditional) distribution is being considered. For example, $p(y|x)$ denotes $p_{Y|X}(y|x)$. The j -th user in the i -th cell is indexed as ij , where $i \in \{a, b\}$, $j \in \Theta_i \triangleq \{0, \dots, (K_i - 1)\}$ and K_i is the user-number in cell- i . Hence the j -th transmitter in the i -th cell is denoted as $\text{Tx}ij$, whose message, input random variable, and rate are denoted as M_{ij} , X_{ij} and R_{ij} , respectively. The signal-to-noise ratio (SNR) and interference-to-noise ratio (INR) from $\text{Tx}ij$ and $\text{Tx}i'j$ to $\text{Rx}i$ are written as SNR_{iji} and $\text{INR}_{i'ji}$, respectively, where $i, i' \in \{a, b\}$ and $i \neq i'$. Similar notation is used for the

¹Since we are studying a more general channel model here than [22], the missing proofs in [22] can be completed by setting $K_a = 2$ and $K_b = 1$ in the results and proofs in Section III and Appendices. Typographical errors in [22] are also corrected in this version.

other cross-cell terms.

The K_i -tuples of messages, inputs and rates of cell i as denoted as M_i , X_i and R_i . For example, the vector of inputs in cell- a ($X_{a0}, \dots, X_{a(K_a-1)}$) are denoted as X_a , with similar abbreviation used for M_a and R_a . However, the Gaussian noise Z_i , output Y_i and random variables S_i and T_i are just single random variables. As defined later, we use \mathcal{T}_i to denote a subset of user indices in cell i that necessarily contain the interfering user $i0$, and Ω_i to denote an arbitrary subset of users in cell i . The sets $\bar{\mathcal{T}}_i$ and $\bar{\Omega}_i$ are defined as the complements of \mathcal{T}_i and Ω_i . Again, the inputs of users in \mathcal{T}_i or Ω_i are written as $X_{\mathcal{T}_i}$ and X_{Ω_i} . But the same rule does not apply to $R_{\mathcal{T}_i}$ and R_{Ω_i} , which denote $\sum_{ij \in \mathcal{T}_i} R_{ij}$ and $\sum_{ij \in \Omega_i} R_{ij}$. We use \mathbb{C} to denote the set of complex numbers, $X \sim \mathcal{CN}(0, \sigma^2)$ to denote a complex random variable X whose distribution is the complex circularly symmetric zero-mean Gaussian distribution with variance σ^2 , and $|\cdot|$ to represent the magnitude of a complex number.

To avoid writing out the rate regions explicitly all the time, we heavily rely on a generic region, which will be defined in Section II.D. The generic region is described by four set functions, each of which is a generic function of some subset \mathcal{T}_i or Ω_i , and will be further defined in various concrete contexts whenever a region needs to be specified.

The rest of the paper is organized as follows. Section II introduces the three channel models of MAC-IC-MAC and formulates the problem. Section III presents the five main results listed above; Section VI concludes the paper. Some detailed proofs are relegated to the appendices.

II. CHANNEL MODELS AND PROBLEM FORMULATION

In this section, we introduce in sequence the general discrete memoryless MAC-IC-MAC, the semi-deterministic MAC-IC-MAC and the Gaussian MAC-IC-MAC. We also define a generic region which we will use repeatedly in subsequent sections to describe inner and outer bounds on the capacity region of those three MAC-IC-MAC models.

A. Discrete Memoryless MAC-IC-MAC (DM MAC-IC-MAC)

In a DM MAC-IC-MAC, as shown in Fig. 3, there are two uplink communication cells: ($\text{Tx}a0 \dots \text{Tx}a(K_a - 1) \rightarrow \text{Rx}a$) and ($\text{Tx}b0 \dots \text{Tx}b(K_b - 1) \rightarrow \text{Rx}b$). Two interference links exist between these two cells which are governed by the marginal probability $p_{Y_a|X_{b0}}(y_a|x_{b0})$ and $p_{Y_b|X_{a0}}(y_b|x_{a0})$, respectively. The definition of DM MAC-IC-MAC is given below. (Please recall the notation system we have introduced in Section I, such as $X_i = \{X_{i0}, \dots, X_{i(K_i-1)}\}$ and etc.)

Definition 1. A (K_a, K_b) discrete memoryless MAC-IC-MAC is a $(K_a + K_b)$ -transmitter-2-receiver network with transition probability satisfying

$$p(y_i|x_i, x_{i'}) = p(y_i|x_i, x_{i'0}) \quad (1)$$

for $i, i' \in \{a, b\}$ and $i \neq i'$. The input and output random variables X_{ij} and Y_i are taken from discrete alphabet sets \mathcal{X}_{ij} and \mathcal{Y}_j respectively, where $j \in \{0, \dots, K_i\}$. Message M_{ij} is generated from set \mathcal{M}_{ij} uniformly at random, and encoded at transmitters $\text{Tx}ij$. Receiver $\text{Rx}i$ decodes M_i as \hat{M}_i .

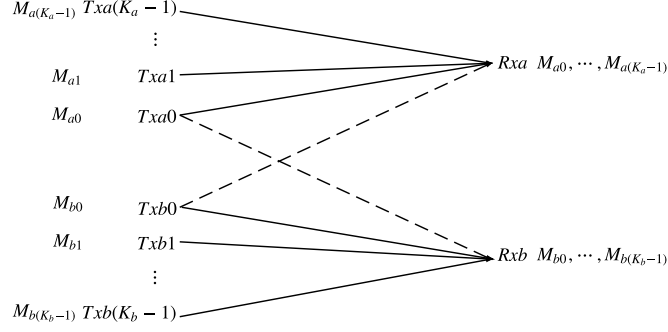


Fig. 3. Discrete memoryless MAC-IC-MAC

Given the channel as defined in Definition 1, a $(n, R_a, R_b, P_e^{(n)})$ coding scheme for a DM MAC-IC-MAC consists of

- M_{ij} , the message of transmitter ij , assumed to be uniformly distributed over $\mathcal{M}_{ij} \in \{1, \dots, 2^{nR_{ij}}\}$;
- encoding functions $f_{ij}(\cdot)$ such that,

$$f_{ij}(\cdot) : \mathcal{M}_{ij} \mapsto \mathcal{X}_{ij}^n, m_{ij} \mapsto x_{ij}^n(m_{ij})$$

- decoding functions $g_i(\cdot)$ such that

$$g_i(\cdot) : \mathcal{Y}_i^n \mapsto \prod_{j=0}^{K_i-1} \mathcal{M}_{ij}, y_i^n \mapsto \hat{m}_i$$

The probability of error $P_e^{(n)}$ is defined to be

$$P_e^{(n)} = P \left\{ M_i \neq \tilde{M}_i : i \in \{a, b\} \right\}$$

A $K_a + K_b$ -tuple rate (R_a, R_b) is said to be achievable if there exists a sequence of $(n, R_a, R_b, P_e^{(n)})$ coding schemes for which $P_e^{(n)} \rightarrow 0$ as $n \rightarrow \infty$.

The capacity region of the MAC-IC-MAC, denoted as \mathcal{C} , is the closure of achievable rate-tuples. An inner bound for \mathcal{C} is obtained in Section III.A.

Remark 2. The study of the MAC-IC-MAC at high SNR is also relevant to the fully-connected $(K_a + K_b)$ -transmitter, 2-receiver, two-cell interference networks when the interference from all but one transmitter in each cell are sufficiently weak so that they are received at the unintended receiver at the noise level (and are therefore treated as noise).

B. Semi-Deterministic MAC-IC-MAC

The received interference in a semi-deterministic MAC-IC-MAC (as shown in Fig. 4) has a special structure as defined below. This structure includes specialization to Gaussian MAC-IC-MACs. Moreover, we will see that the genie-aided arguments of [2] for the 2-user IC can be extended to the semi-deterministic MAC-IC-MAC in obtaining outer bounds to the capacity region.

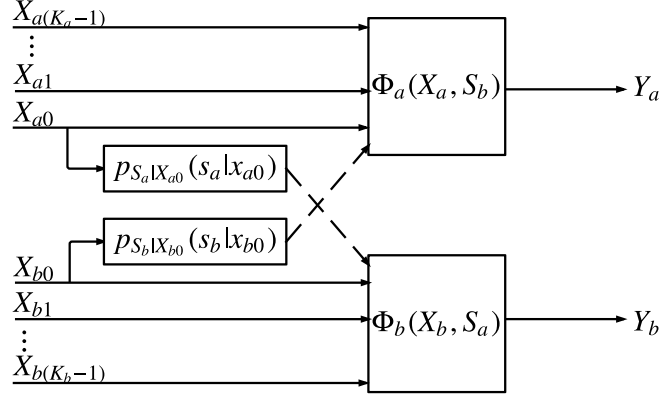


Fig. 4. Semi-deterministic MAC-IC-MAC

Definition 3. Let S_i be a random variable over the alphabet \mathcal{S}_i for $i \in \{a, b\}$. A MAC-IC-MAC is semi-deterministic if the outputs Y_i satisfy

$$Y_i = \Phi_i(X_i, S_{i'}) \quad (2)$$

where for any fixed inputs x_i , the mapping

$$\Phi_i(x_i, \cdot) : \mathcal{S}_{i'} \rightarrow \mathcal{Y}_i, \quad s_{i'} \rightarrow y_i$$

is invertible.

We use \mathcal{C}^{SD} to denote the capacity region of semi-deterministic MAC-IC-MAC. An outer bound will be obtained for \mathcal{C}^{SD} in Section III.B.

C. Gaussian MAC-IC-MAC

Consider an additive Gaussian MAC-IC-MAC, whose input-output relation can be written as

$$Y_a = \sum_{j=0}^{K_a-1} h_{aj,a} X_{aj} + h_{b0,a} X_{b0} + Z_a \quad (3)$$

$$Y_b = \sum_{j=0}^{K_b-1} h_{bj,b} X_{bj} + h_{a0,b} X_{a0} + Z_b \quad (4)$$

where X_{ij} and Y_i are complex input and output random variables, $h_{ij,i}, h_{ij,i'} \in \mathbb{C}$ are path attenuations from Tx ij to Rx i and Rx i' , respectively, and $Z_i \in \mathcal{CN}(0, 1)$ is the additive noise. The transmitted codeword $x_{ij}^n(m_{ij}) \in \mathcal{X}_{ij}^n$ at Tx ij should meet the average per-codeword power constraints:

$$\frac{1}{n} \sum_{t=1}^n |x_{ij,t}(m_{ij})|^2 \leq 1 \quad (5)$$

The SNRs and INR at receiver Rx i are defined to be

$$\text{SNR}_{ij,i} = P_{ij} |h_{ij,i}|^2 \quad (6)$$

$$\text{INR}_{i'0,i} = P_{i'0} |h_{i'0,i}|^2 \quad (7)$$

where P_{ij} is the maximum average transmission power at $\text{Tx}ij$. We denote the capacity region of Gaussian MAC-IC-MAC by \mathcal{C}^G . The Gaussian MAC-IC-MAC is a special case of the semi-deterministic MAC-IC-MAC, as can be seen by choosing S_i in (2) to be $S_i = h_{i'0,i}X_{i'0} + Z_i$. In Section III.D, we will get an inner bound and an outer bound for \mathcal{C}^G . The inner bound will be derived based on an explicit coding scheme, and these two bounds will be within a one-bit gap.

D. A Generic Region

For brevity, we provide a definition of a generic region in this subsection in terms of which all key results are succinctly expressed later in the paper.

Definition 4. For any sets $\Omega_i \subseteq \{i0, \dots, i(K_i - 1)\}$, $\Upsilon_i' \subseteq \{i1, \dots, i(K_i - 1)\}$ and $\Upsilon_i = \Upsilon_i' \cup \{i0\}$, where $i \in \{a, b\}$, let \mathcal{A}_{Υ_i} and \mathcal{E}_{Υ_i} be real-valued functions of set Υ_i , and \mathcal{B}_{Ω_i} and \mathcal{G}_{Ω_i} be real-valued functions of the set Ω_i . Define $\mathcal{R}(R_a, R_b, \mathcal{A}, \mathcal{B}, \mathcal{E}, \mathcal{G}) \subseteq \mathbb{R}_+^{K_a+K_b}$ to be the region of (R_a, R_b) such that for any Ω_i and Υ_i ,

$$R_{\Omega_a} \leq \mathcal{B}_{\Omega_a} \quad (8)$$

$$R_{\Omega_b} \leq \mathcal{B}_{\Omega_b} \quad (9)$$

$$R_{\Upsilon_a} + R_{\Omega_b} \leq \mathcal{A}_{\Upsilon_a} + \mathcal{G}_{\Omega_b} \quad (10)$$

$$R_{\Omega_a} + R_{\Upsilon_b} \leq \mathcal{A}_{\Upsilon_b} + \mathcal{G}_{\Omega_a} \quad (11)$$

$$R_{\Upsilon_a} + R_{\Upsilon_b} \leq \mathcal{E}_{\Upsilon_a} + \mathcal{E}_{\Upsilon_b} \quad (12)$$

$$R_{\Upsilon_a} + R_{\Omega_a} + R_{\Upsilon_b} \leq \mathcal{A}_{\Upsilon_a} + \mathcal{G}_{\Omega_a} + \mathcal{E}_{\Upsilon_b} \quad (13)$$

$$R_{\Upsilon_a} + R_{\Omega_b} + R_{\Upsilon_b} \leq \mathcal{A}_{\Upsilon_b} + \mathcal{G}_{\Omega_b} + \mathcal{E}_{\Upsilon_a} \quad (14)$$

As will be observed in the later section, each set function represents a mutual information term for a certain set Ω_i or Υ_i . However, the exact mapping of these set functions are not given here. Instead, they will be assigned according to the channel model – DM, semi-deterministic or Gaussian – and as to whether the bound under consideration is an inner bound or an outer bound. When a concrete region needs to be shown, we will specialize each set function, and a rate region can be described by replacing the generic set functions in inequalities (8)-(14) with the specified ones.

III. MAIN RESULTS

In the previous section, we introduced several different models of MAC-IC-MAC. This section presents the main results of this paper, which are inner and/or outer bounds on the capacity regions for those models. Section III.A states an inner bound on the capacity region of general DM MAC-IC-MAC, supported by the random coding framework. Section III.B presents an outer bound for the semi-deterministic MAC-IC-MAC using a genie-aided decoding argument, and this outer bound will be proved, in Section III.C, to be within quantifiable gap to the inner bound derived in Section III.A. Section III.D focuses on the Gaussian MAC-IC-MAC, where we give a single

region inner bound (rather than a convex hull of a union of regions) of its capacity region based on the Etkin-Tse-Wang-type coding scheme [3], and a single region outer bound. These two bounds are within a one-bit gap. Finally, in Section III.E, the discussion on the GDoF of the Gaussian MAC-IC-MAC reveals how the non-interfering transmitters improve the symmetric sum GDoF under various interference strength parameter α , defined as the ratio of the INR to SNR, both expressed in dB scale, at high SNR.

A. Inner Bound on the Capacity Region of DM MAC-IC-MAC

As being mentioned earlier, the H-K scheme (and its alternative CMG scheme) lead to the best inner bounds to date for the interference channel. In this section, we apply the CMG type scheme to the MAC-IC-MAC. More specifically, we employ superposition coding at Tx i 0 and use a single-user random codebook for transmitter Tx i j , $j \neq 0$. Applying the joint-typicality decoding argument we derive a CMG-like inner bound for the DM MAC-IC-MAC as following.

Definition 5. For any given jointly distributed random variables $(Q, U_{a0}, X_a, U_{b0}, X_b)$ with distribution $p(q)p(u_{a0}, x_a|q)p(u_{b0}, x_b|q)$, let the non-negative real valued set functions A, B, E and G be

$$A_{\mathcal{R}_i} \triangleq I(X_{\mathcal{R}_i}; Y_i | X_{\bar{\mathcal{R}}_i}, U_{i0}, U_{i'0}, Q) \quad (15)$$

$$B_{\Omega_i} \triangleq I(X_{\Omega_i}; Y_i | X_{\bar{\Omega}_i}, U_{i'0}, Q) \quad (16)$$

$$E_{\mathcal{R}_i} \triangleq I(X_{\mathcal{R}_i}, U_{i'0}; Y_i | X_{\bar{\mathcal{R}}_i}, U_{i0}, Q) \quad (17)$$

$$G_{\Omega_i} \triangleq I(X_{\Omega_i}, U_{i'0}; Y_i | X_{\bar{\Omega}_i}, Q) \quad (18)$$

and the region $\mathcal{R}_{\text{in}}(Q, U_{a0}, X_a, U_{b0}, X_b) \subset \mathbb{R}_+^{K_a+K_b}$ to be

$$\mathcal{R}_{\text{in}}(Q, U_{a0}, X_a, U_{b0}, X_b) \triangleq \mathcal{R}(R_a, R_b, A, B, E, G) \quad (19)$$

Theorem 6. For a (K_a, K_b) DM MAC-IC-MAC,

$$\begin{aligned} \mathcal{R}_{\text{in}} &\triangleq \bigcup_{(X_a, U_{a0}) - \circ - Q - \circ - (X_b, U_{b0})} \mathcal{R}_{\text{in}}(Q, U_{a0}, X_a, U_{b0}, X_b) \\ &\subseteq \mathcal{C} \end{aligned}$$

Proof: The proof is done in two parts. With the interfering transmitters employing the CMG rate-splitting and superposition coding scheme as in the two-user interference channel in [1], and the non-interfering transmitters using independent random coding (as in a MAC), with each receiver decoding all its desired messages and the common sub-message of the interfering transmitter using simultaneous non-unique decoding, standard random analysis techniques lead to a intermediate rate region containing two auxiliary random variables which represent the common sub-message or “cloud” codewords of interfering transmitters, and two auxiliary rates corresponding to these codebooks. This part of the proof extends the reliability analysis of the CMG coding scheme [1] to the MAC-IC-MAC. More specifically: 1) The non-interfering receiver Tx i j , $j \neq 0$ sends information m_{ij}^n by some codeword $x_{ij}^n(m_{ij}^n)$ using a single-user random codebook; 2) the interfering transmitter Tx i 0 splits its message m_{i0}

into common and private sub-messages $m_{i0,c}$ and $m_{i0,p}$, respectively. The common information is first encoded into the cloud center codeword $u_{i0}^n(m_{i0,c})$, and then, based on the private sub-message $m_{i0,p}$, the message m_{i0} is encoded into the codeword $x_{i0}^n(u_{i0}^n(m_{i0,c}), m_{i0,p})$ for transmission; 3) Rx i decodes all the information m_{ij}^n , $j \neq 0$, and the public and private information $m_{i0,c}$ and $m_{i0,p}$ from all of its intended transmitters, and the common information $m_{i'0,c}$ from the non-intended transmitter Tx $i'0$ (but non-uniquely).

Second, we carry out Fourier-Motzkin elimination of the two auxiliary rates analytically to project the rate region onto $\mathbb{R}_+^{K_a+K_b}$ of message rates, despite the numbers of users in both cells being arbitrary. In this step, the algebraic structure of the inequality system created in the first step is utilized, since a hand-crafted, brute-force Fourier-Motzkin elimination is not possible. The detailed proof is given in Appendix A. ■

B. An Outer Bound for the Semi-Deterministic MAC-IC-MAC

Next, we focus our attention on the semi-deterministic MAC-IC-MAC. The inner bound stated in the previous subsection automatically applies to the semi-deterministic MAC-IC-MAC. Besides, the semi-deterministic property given in (2) enables us to determine an outer bound. The key idea is to allow a genie to pass some channel side information about S_i to help Rx i 's decoding. Since providing side information to the receiver will not decrease the channel capacity, an outer bound can therefore be characterized. This idea was first introduced in the work of [2] for the two-user IC, and we show a similar argument that can be applied to semi-deterministic MAC-IC-MAC too.

Definition 7. Given any set of random variables (Q, X_a, T_a, X_b, T_b) with joint distribution $p(q)p(x_a, t_a|q)p(x_b, t_b|q)$, let the set functions \bar{A} , \bar{B} , \bar{E} and \bar{G} be

$$\bar{A}_{\Upsilon_i} \triangleq H(Y_i|X_{\bar{\Upsilon}_i}, T_i, X_{i'0}, Q) - H(S_{i'}|X_{i'}, Q) \quad (20)$$

$$\bar{B}_{\Omega_i} \triangleq H(Y_i|X_{\bar{\Omega}_i}, X_{i'0}, Q) - H(S_{i'}|X_{i'}, Q) \quad (21)$$

$$\bar{E}_{\Upsilon_i} \triangleq H(Y_i|X_{\bar{\Upsilon}_i}, T_i, Q) - H(S_{i'}|X_{i'}, Q) \quad (22)$$

$$\bar{G}_{\Omega_i} \triangleq H(Y_i|X_{\bar{\Omega}_i}, Q) - H(S_{i'}|X_{i'}, Q) \quad (23)$$

where the auxiliary random variable T_i takes value from \mathcal{S}_i according to

$$p_{T_i|Q, X_i, X_{i'}}(t_i|q, x_i, x_{i'}) \triangleq p_{S_i|X_{i0}}(t_i|x_{i0}) \quad (24)$$

and is conditionally independent of S_i given X_i . Define the region $\mathcal{R}_{\text{out}}^{\text{SD}}(Q, T_a, X_a, T_b, X_b) \subset \mathbb{R}_+^{K_a+K_b}$ to be $\mathcal{R}(R_a, R_b, \bar{A}, \bar{B}, \bar{E}, \bar{G})$.

Remark 8. According to (24), a genie informs Rx i the distribution of the side information S_i (refer to Fig. (4)), but not S_i itself, since S_i and T_i are conditionally independent on X_i . Therefore we have $H(T_i|X_{i0}) = H(S_i|X_{i0})$ and $H(T_i^n) = H(S_i^n)$. This observation plays an important role in the proof of the following theorem.

Theorem 9. For semi-deterministic MAC-IC-MAC,

$$\begin{aligned} \mathcal{C}^{\text{SD}} &\subseteq \mathcal{R}_{\text{out}}^{\text{SD}} \\ &\triangleq \bigcup_{(X_a, T_a) - \circ - Q - \circ - (X_b, T_b)} \mathcal{R}_{\text{out}}^{\text{SD}}(Q, T_a, X_a, T_b, X_b) \end{aligned}$$

Proof: We first extend the genie-aided argument in [2] to upper bound the sum rates for a given user subset \mathcal{Y}_i or Ω_i . Then, linearly combining these bounds to cancel the genie random variables yields the theorem. Please refer to Appendix B for details. ■

C. Quantifiable Gap

Next, we show that for the semi-deterministic MAC-IC-MAC, the outer bound obtained in Theorem (9) is within a quantifiable gap of its capacity region. To prove this, we construct a sub-region inside the inner bound in Theorem 6 so the gap between this sub-region and the outer region can be bounded.

Theorem 10. For any rate tuple $(R_a, R_b) \in \mathcal{R}_{\text{out}}^{\text{SD}}$, we have

$$(R_a - I(X_{b0}; S_b | T_b) \mathbf{1}_{1 \times K_a}, R_b - I(X_{a0}; S_a | T_a) \mathbf{1}_{1 \times K_b}) \in \mathcal{R}_{\text{in}}$$

where $\mathbf{1}_{1 \times K_i}$ is the all-one row vector of dimension K_i .

Proof: For some input distribution $p(q)p(u_{a0}, x_a | q)p(u_{b0}, x_b | q)$, we first construct a sub-region of the inner region $\mathcal{R}_{\text{in}}^{\text{SD}}(Q, U_{a0}, X_a, U_{b0}, X_b) \subseteq \mathcal{R}_{\text{in}}^{\text{SD}}(Q, U_{a0}, X_a, U_{b0}, X_b)$, and then choose U_{i0} in $\mathcal{R}_{\text{in}}^{\text{SD}}(Q, U_{a0}, X_a, U_{b0}, X_b)$ to be identically distributed to T_i , and obtain a sub-region $\mathcal{R}_{\text{in}}^{\prime\prime\text{SD}}(Q, T_{a0}, X_a, T_{b0}, X_b) \subseteq \mathcal{R}_{\text{in}}^{\text{SD}}(Q, U_{a0}, X_a, U_{b0}, X_b)$. Finally, it can be shown that the gap between $\mathcal{R}_{\text{in}}^{\prime\prime\text{SD}}(Q, T_{a0}, X_a, T_{b0}, X_b)$ and $\mathcal{R}_{\text{out}}^{\text{SD}}(Q, T_{a0}, X_a, T_{b0}, X_b)$ is quantifiable, so that the gap between $\mathcal{R}_{\text{in}}^{\text{SD}}$ and $\mathcal{R}_{\text{out}}^{\text{SD}}$ is also quantifiable. We relegate the complete proof to Appendix C. ■

D. Approximate Capacity Region of Gaussian MAC-IC-MAC within One Bit

The quantifiable gap on semi-deterministic MAC-IC-MAC capacity directly yields an approximation of the capacity region to within a constant-bit-gap for the Gaussian MAC-IC-MAC. The bounds in Theorem 6 and 9 are sufficient to provide an approximate capacity region for Gaussian MAC-IC-MAC, and according to Theorem (10), the gap between the inner and outer bounds will not exceed one bit. To verify the gap, choose $T_i = h_{i0, i'} X_{i0} + Z_{i'}$ and $Z_{i'} \perp Z_{i'}$ so that

$$\begin{aligned} I(X_{i0}; S_i | T_i) &= h(h_{i0, i'} X_{i0} + Z_{i'} | h_{i0, i'} X_{i0} + Z_{i'}) \\ &\quad - h(h_{i0, i'} X_{i0} + Z_{i'} | X_{i0}, h_{i0, i'} X_{i0} + Z_{i'}) \\ &\leq h(Z_{i'} - Z_{i'}) - h(Z_{i'}) \\ &= 1 \text{ bit} \end{aligned}$$

However, the inner bound for DM MAC-IC-MAC in Theorem 6 is a union of bounds, each of which corresponds to some choice of joint distribution $p(q)p(u_a, x_a | q)p(u_b, x_b | q)$. A simple *explicit* coding scheme is preferred whose

achievable rate region is within a constant gap of the capacity region for Gaussian MAC-IC-MAC. Here, we will show that the Etkin-Tse-Wang type coding scheme [3] at the interfering transmitters and Gaussian codebooks (with maximum allowable power) at the non-interfering transmitters will lead to an inner bound within one bit gap to the capacity.

The available power at the interfering transmitters is split between the common and private message in a manner such that the private message arrives at the unintended receiver at the noise level.

Definition 11. Suppose $C(P) = \log(1 + P)$ for some $P \geq 0$. Define the coefficient

$$\mu_i \triangleq \min \left\{ 1, \frac{1}{\text{INR}_{i'0,i}} \right\}, \quad (25)$$

set functions A , B , E and G be

$$A_{\mathcal{T}_i} \triangleq C \left(\frac{\mu_i \text{SNR}_{i0,i} + \sum_{ij \in \mathcal{T}_i \setminus \{i0\}} \text{SNR}_{ij,i}}{1 + \mu_{i'} \text{INR}_{i'0,i}} \right) \quad (26)$$

$$B_{\Omega_i} \triangleq C \left(\frac{\sum_{ij \in \Omega_i} \text{SNR}_{ij,i}}{1 + \mu_{i'} \text{INR}_{i'0,i}} \right) \quad (27)$$

$$E_{\mathcal{T}_i} \triangleq C \left(\frac{\mu_i \text{SNR}_{i0,i} + \sum_{ij \in \mathcal{T}_i \setminus \{i0\}} \text{SNR}_{ij,i}}{1 + \mu_{i'} \text{INR}_{i'0,i}} \right. \\ \left. + \frac{(1 - \mu_{i'}) \text{INR}_{i'0,i}}{1 + \mu_{i'} \text{INR}_{i'0,i}} \right) \quad (28)$$

$$G_{\Omega_i} \triangleq C \left(\frac{\sum_{ij \in \Omega_i} \text{SNR}_{ij,i} + \text{INR}_{i'0,i}}{1 + \mu_{i'} \text{INR}_{i'0,i}} \right) \quad (29)$$

and the region $\mathcal{R}_{\text{in}}^G \triangleq \mathcal{R}(R_a, R_b, A, B, E, G)$.

Theorem 12. For the Gaussian MAC-IC-MAC, $\mathcal{R}_{\text{in}}^G \subseteq \mathcal{C}^G$.

Proof: The result can be verified by computing the mutual information terms in Theorem 6 in the context of the proposed coding scheme. First, no time-sharing is adopted between any transmitters. $\text{Tx}ij$, $j \neq 0$, uses single-user Gaussian codebook, while $\text{Tx}i0$ employs Gaussian superposition coding in terms of splitting transmission power between each of their common and public messages $P_{i0,c}$ and $P_{i0,p}$. If $\text{INR}_{i'0,i} < 1$, the Gaussian noise dominates the interference at $\text{Rx}i$ from $\text{Tx}i'0$. Thus, $\text{Tx}i'0$ allocates all the transmission power to its private message, and $\text{Rx}i$ treats the interference from $\text{Tx}i'$ as noise. When $\text{INR}_{i'0,i} \geq 1$, the transmission power at $\text{Tx}i0$ needs to be split so that the power of the unintended private messages from $\text{Tx}i'$ arrives at $\text{Rx}i$ at unity (normalized to), i.e.

$$P_{i0,p} \sim \mathcal{CN}(0, \frac{1}{|h_{i0,i'}|^2}) \quad (30)$$

Computing all the set functions in Theorem 6 in the context of this coding strategy, the Theorem is proved. \blacksquare

Because the Gaussian MAC-IC-MAC is semi-deterministic (where $S_i = h_{i0,i'} X_{ij} + Z_{i'}$), Theorem 9 is applicable, and an outer bound on the capacity region can be characterized accordingly.

Definition 13. Let the set functions \bar{A} , \bar{B} , \bar{E} and \bar{G} be,

$$\bar{A}_{\mathcal{I}_i} \triangleq \mathbb{C} \left(\sum_{ij \in \mathcal{I}_i \setminus \{i0\}} \text{SNR}_{ij,i} + \frac{\text{SNR}_{i0,i}}{1 + \text{INR}_{i0,i'}} \right) \quad (31)$$

$$\bar{B}_{\Omega_i} \triangleq \mathbb{C} \left(\sum_{ij \in \Omega_i} \text{SNR}_{ij,i} \right) \quad (32)$$

$$\bar{E}_{\mathcal{I}_i} \triangleq \mathbb{C} \left(\sum_{ij \in \mathcal{I}_i \setminus \{i0\}} \text{SNR}_{ij,i} + \frac{\text{SNR}_{i0,i}}{1 + \text{INR}_{i0,i'}} + \text{INR}_{i'0,i} \right) \quad (33)$$

$$\bar{G}_{\Omega_i} \triangleq \mathbb{C} \left(\sum_{ij \in \Omega_i} \text{SNR}_{ij,i} + \text{INR}_{i'0,i} \right). \quad (34)$$

and define $\mathcal{R}_{\text{out}}^{\text{G}} \triangleq \mathcal{R}(R_a, R_b, \bar{A}, \bar{B}, \bar{E}, \bar{G})$.

Theorem 14. For Gaussian MAC-IC-MAC, $\mathcal{C}^{\text{G}} \subseteq \mathcal{R}_{\text{out}}^{\text{G}}$.

Proof: Refer to Appendix D. ■

Next we show that the gap between $\mathcal{R}_{\text{in}}^{\text{G}}$ and $\mathcal{R}_{\text{out}}^{\text{G}}$ is within one bit, which implies an explicit coding scheme ensures an inner bound within one bit gap to the capacity.

Theorem 15. There is no more than a one bit gap between $\mathcal{R}_{\text{in}}^{\text{G}}$ and $\mathcal{R}_{\text{out}}^{\text{G}}$.

Proof: It is enough to show the gap between each pattern is within one bit. We bound the gap between $E_{\mathcal{I}_i}$ and $\bar{E}_{\mathcal{I}_i}$ for instance.

$$\begin{aligned} E_{\mathcal{I}_i} &= \log \left(2 + \mu_i \text{SNR}_{i0,i} + \sum_{ij \in \mathcal{I}_i \setminus \{i0\}} \text{SNR}_{ij,i} \right. \\ &\quad \left. + (1 - \mu_{i'}) \text{INR}_{i'0,i} \right) - \log \left(1 + \mu_{i'} \text{INR}_{i'0,i} \right) \\ &\geq \log \left(2 + \mu_i \text{SNR}_{i0,i} + \sum_{ij \in \mathcal{I}_i \setminus \{i0\}} \text{SNR}_{ij,i} \right. \\ &\quad \left. + (1 - \mu_{i'}) \text{INR}_{i'0,i} \right) - 1 \\ &= \log \left(2 + \min \left\{ 1, \frac{1}{\text{INR}_{i0,i'}} \right\} \text{SNR}_{i0,i} + \text{INR}_{i'0,i} \right. \\ &\quad \left. + \sum_{ij \in \mathcal{I}_i \setminus \{i0\}} \text{SNR}_{ij,i} - \min \left\{ \text{INR}_{i'0,i}, 1 \right\} \right) - 1 \\ &\geq \mathbb{C} \left(\sum_{ij \in \mathcal{I}_i \setminus \{i0\}} \text{SNR}_{ij,i} + \frac{\text{SNR}_{i0,i}}{1 + \text{INR}_{i0,i'}} + \text{INR}_{i'0,i} \right) - 1 \\ &= \bar{E}_{\mathcal{I}_i} - 1 \end{aligned}$$

Other gaps can be verified similarly. ■

E. Generalized Degrees of Freedom Region of Gaussian MAC-IC-MAC

The generalized degrees of freedom (GDoF) is an information-theoretic metric that characterizes the simultaneously accessible signal-level dimensions (per channel use) by the users of a network in the limit of high SNR, while the ratios of the SNRs and INRs relative to a reference SNR, each expressed in the dB scale, are held constant, with each constant taken, in the most general case, to be arbitrary. In this section, we first compute the GDoF region of the Gaussian MAC-IC-MAC and then focus on the study of the symmetric GDoF curve.

Given the Gaussian MAC-IC-MAC model as defined by (3) and (4), let ρ be a nominal value for SNR or INR, define $\bar{\alpha} = (\alpha_{a0,a}, \dots, \alpha_{a(K_a-1),a}, \alpha_{b0,b}, \dots, \alpha_{b(K_b-1),b}, \alpha_{a0b}, \alpha_{b0a})$ as

$$\alpha_{ij,i} = \frac{\log(\text{SNR}_{ij,i})}{\log \rho} \quad \alpha_{i0,i'} = \frac{\log(\text{INR}_{i0,i'})}{\log \rho}$$

Remark 16. The GDoF of the Gaussian MAC-IC-MAC applies also to a fully connected $(K_a + K_b)$ -transmitter-2-receiver interference network, when the interference from all but one transmitter in each cell is sufficiently weak to be at the noise level, i.e., the corresponding INR exponents of these links are equal to zero.

Definition 17. The *generalized degrees of freedom region* of a (K_a, K_b) Gaussian MAC-IC-MAC $\mathcal{D}(K_a, K_b, \bar{\alpha}) \in R_+^{K_a+K_b}$ of the capacity region $\mathcal{C}^G(K_a, K_b, \bar{\alpha})$ is defined as

$$\begin{aligned} \{(d_a, d_b) : d_{ij} &= \lim_{\rho \rightarrow \infty} \frac{R_{ij}}{\log \rho}, i \in \{a, b\} \\ j &\in \{0, \dots, K_i - 1\}, \\ \text{and } (R_a, R_b) &\in \mathcal{C}^G(K_a, K_b, \bar{\alpha})\} \end{aligned} \quad (35)$$

Since we have the capacity region to within a one-bit gap for the Gaussian MAC-IC-MAC, computing GDoF region can be easily done by substituting $\text{SNR}_{ij,i} = \rho^{\alpha_{ij,i}}$ and $\text{INR}_{ij,i'} = \rho^{\alpha_{ij,i'}}$ in (31)-(34), and computing the limits of the right hand side terms under $\rho \rightarrow \infty$.

Definition 18. For any two real numbers x and y , let $(x - y)^+ \triangleq \max\{0, x - y\}$. Define the set functions a , b , e and g as

$$a_{\mathcal{Y}_i} = \max \left\{ \max_{ij \in \mathcal{Y}_i \setminus \{i0\}} \alpha_{ij,i}, (\alpha_{i0,i} - \alpha_{i0,i'})^+ \right\} \quad (36)$$

$$b_{\Omega_i} = \max_{ij \in \Omega_i} \alpha_{ij,i} \quad (37)$$

$$e_{\mathcal{Y}_i} = \max \left\{ \max_{ij \in \mathcal{Y}_i \setminus \{i0\}} \alpha_{ij,i}, (\alpha_{i0,i} - \alpha_{i0,i'})^+, \alpha_{i'0i} \right\} \quad (38)$$

$$g_{\Omega_i} = \max \left\{ \max_{ij \in \Omega_i} \alpha_{ij,i}, \alpha_{i'0i} \right\} \quad (39)$$

Theorem 19. For (K_a, K_b) Gaussian MAC-IC-MAC, the GDoF region $\mathcal{D}(K_a, K_b, \bar{\alpha})$ is $\mathcal{R}(d_a, d_b, a, b, e, g)$.

A Gaussian MAC-IC-MAC is symmetric if $\text{SNR}_{ij,i} = \text{SNR}$ and $\text{INR}_{i'0,i} = \text{INR}$, i.e. all SNRs are identical and all INRs are identical. This does not mean the channel attenuation $h_{ij,i}$ s or $h_{i'0,i}$ s are identical, since transmitters

may choose different transmission power. For arbitrary i and j , SNR and INR terms can thus be normalized as

$$\text{INR} = \text{SNR}^\alpha = \rho^\alpha$$

which implies $\alpha_{ij,i} = 1$ and $\alpha_{i0,i'} = \alpha$. For symmetric Gaussian MAC-IC-MAC, a simplified, yet instructive metric, the so-called *symmetric GDoF*, which is only a function of K and the interference strength parameter α , can be defined as follows.

Definition 20. For a symmetric K -user Gaussian MAC-IC-MAC with GDoF region $\mathcal{D}_{\text{sym}}^G(K, \bar{\alpha})$, the *symmetric generalized degree-of-freedom* $d_{\text{sym}}(K, \alpha)$ is defined as the solution to the following equation

$$d_{\text{sym}}(K, \alpha) \triangleq \max_{\substack{d=d_{a0}=\dots=d_{a(K_a-1)}=d_{b0}=\dots=d_{b(K_b-1)} \\ (d_a, d_b) \in \mathcal{D}(\bar{\alpha})}} d$$

Given the GDoF region in Theorem 19, the symmetric GDoF of Gaussian MAC-IC-MAC can be derived according to the definition, which is stated below.

Theorem 21. Consider a symmetric K -user Gaussian MAC-IC-MAC where $\alpha_{ij,i} = 1$ and $\alpha_{i0,i'} = \alpha \geq 0$, the symmetric GDoF $d_{\text{sym}}(K, \alpha)$ for $K \geq 2$ is given by

$$d_{\text{sym}}(K, \alpha) = \begin{cases} 1 & 0 \leq \alpha < 1 - \frac{1}{K} \\ -\frac{1}{K+1}\alpha + \frac{2}{K+1} & 1 - \frac{1}{K} \leq \alpha < 1 \\ \frac{1}{K+1}\alpha & 1 \leq \alpha < 1 + \frac{1}{K} \\ 1 & \alpha \geq 1 + \frac{1}{K} \end{cases}$$

The symmetric GDoF curve for general K is plotted in Fig.2 in Section I. In Fig.5, we plot the numerical results of sum symmetric GDoF, i.e. $Kd_{\text{sym}}(K, \alpha)$ for K equals 1, 2, 3 and 4. Sum symmetric GDoF offers us a straightforward way to observe the benefit of adding interference free transmitters to the entire cell GDoF, which is a measure of cell spectrum efficiency.

When $K = 1$, we see the “W” curve which recovers the symmetric GDoF curve for 2-user IC given in [3], since the 2-user IC is a special case of Gaussian MAC-IC-MAC when we place only one (interfering) transmitter in each cell. When $K \geq 2$, we get a “V” curve with flat shoulders on both sides of the curve, which illustrates the benefit of adding non-interfering transmitters to the cell. Indeed, full GDoF (which is 1) can be achieved for $\alpha \in [0, \frac{1}{2}] \cup [\frac{3}{2}, \infty)$. When $\alpha = 1$, the sum symmetric GDoF becomes $\frac{K}{K+1}$, approaching 1 for large K . As K increases, the width of the flat shoulder increases and the width of the dip around $\alpha = 1$ decreases, leading to unit GDoF for an increasing range of α .

Adding even just one non-interfering transmitter to the two-user IC improves GDoF significantly. This can be easily observed at $\alpha = \frac{1}{2}$ in Fig. 5. Because of interference, a two-user IC only allows each user to achieve $\frac{1}{2}$ DoF, while a $(K_a = 2, K_b = 2)$ MAC-IC-MAC could achieve full GDoF while providing $\frac{1}{2}$ DoF to each of the four transmitters, a significant gain.

We explain the benefit of the MAC-IC-MAC over the two-user IC using the deterministic method introduced in [23] by focusing on the achievability of the three key inflection points in the sum GDoF curve, namely, $(1 - \frac{1}{K}, 1)$,

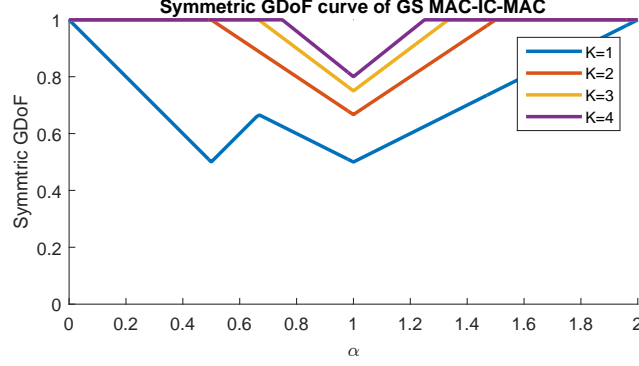


Fig. 5. Sum Symmetric GDoF $Kd_{\text{sym}}(K, \alpha)$ versus α for $K = 1, 2, 3, 4$

$(1, \frac{K}{K+1})$ and $(1 + \frac{1}{K}, 1)$. In what follows, we pick $K = 2$ as an example. Fig. 6 shows the aligned received signals from Rx_i 's perspective for the three key values of α . The horizon line represents the noise floor. The transmitted power is partitioned into multiple levels, such that every power level could carry the same DoF. The shadowed signal levels in the figure are the signal levels being used to transmit information. When $\alpha = \frac{1}{2}$, the lower level of interference $X_{i'0}$ will be under the noise floor, so it would be best to let Tx_i0 and $Tx_{i'}0$ use the lower power level, to get DoF $\frac{1}{2}$ per transmitter and still leave DoF $\frac{1}{2}$ to Tx_i1 and $Tx_{i'}1$. The same principle for when $\alpha = \frac{3}{2}$, in which case the higher lever of $X_{i'0}$ is above the intended received signal, and will be decoded as strong interference at Rx_i . When $\alpha = 1$, Rx_i needs to decode the information from Tx_i0 , Tx_i1 and $Tx_{i'}0$, which gives DoF $\frac{1}{3}$ to each transmitter.

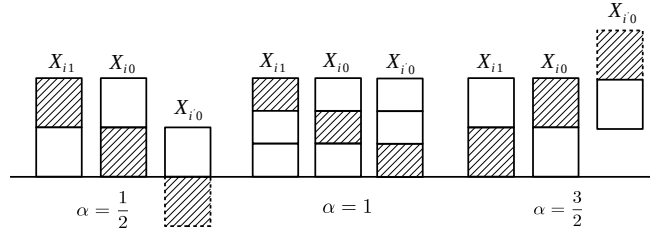


Fig. 6. Achieving symmetric GDoF when $\alpha = \frac{1}{2}$, $\alpha = 1$ and $\alpha = \frac{3}{2}$ for $(2, 2)$ Gaussian MAC-IC-MAC

Remark 22. Initially, the one bit gap result for Gaussian MAC-IC-MAC is obtained with the aid of the Etkin-Tse-Wang type coding scheme, which is different from the coding scheme being discussed here. It is not surprising that two different coding schemes end up with the same symmetric GDoF curve. However, the deterministic method clearly explains the role of the non-interfering transmitters.

Apparently, the MAC-IC-MAC can be viewed as replacing each of the transmitters in a 2-user IC with a group of transmitters, one of which generates interference to the non-intended receiver. Mixing interfering and non-interfering transmitters certainly neutralizes the interference seen at the receivers, which increases the symmetric GDoF per cell. Going back to the example above, it would be interesting to re-explore the achievability using time sharing

scheme. Assume we perform time sharing between $\text{Tx}i0$ and $\text{Tx}i1$ in both cells. For some α , let $d(1, \alpha)$ be the symmetric DoF of the 2-user IC. To share the DoF evenly, $\text{Tx}i0$ has to use $\frac{1}{d(1, \alpha)+1}$ portion of a uniform time duration and leave the rest $\frac{d(1, \alpha)}{d(1, \alpha)+1}$ portion to $\text{Tx}i1$. This way we get sum GDoF $\frac{2d(1, \alpha)}{d(1, \alpha)+1}$ which is less than the fundamental GDoF derived by the optimal coding scheme as shown in Fig. 7, except when α equals 0, 1 and 2. This illustrates the sub-optimality of time sharing in MAC-IC-MAC. The reason why superposition coding outperforms time sharing can be again observed from Fig. 6. Say $\alpha = \frac{1}{2}$, the role of $\text{Tx}i1$ in the optimal scheme is to fill up the unused power levels after $\text{Tx}i0$ instead of sharing DoF with $\text{Tx}i0$.

Time sharing can be performed in another way: let $\text{Txa}0$ and $\text{Txb}1$ send in the first half of unit time, and then $\text{Txa}1$ and $\text{Txb}0$, which is a time sharing scheme between two ZICs. It can be easily verified that this scheme has inferior performance compared to time sharing between non-interfering and interfering transmitters. The symmetric GDoF curve given in Fig. 2 points out the key idea that a linear combination of non-interfering and interfering transmitters does not yield linear combinations of their GDoFs.

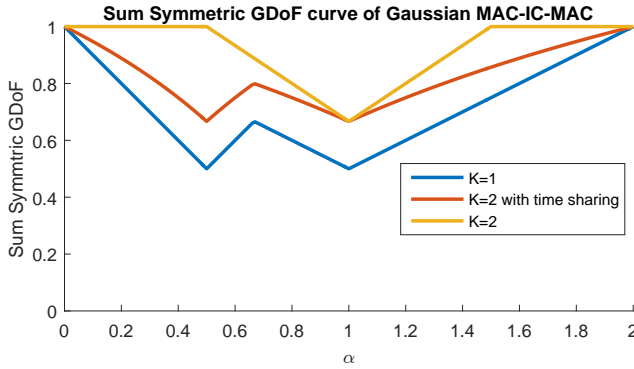


Fig. 7. Superposition coding vs time sharing

IV. CONCLUSIONS

In this work, we extended the framework of Telatar and Tse [2] and Etkin et al [3] to a more general class of networks, namely the MAC-IC-MAC. We characterized the capacity region of semi-deterministic as well as Gaussian MAC-IC-MAC to within a bounded gap (independent of all channel parameters). In particular, it is shown that single-user coding at non-interfering transmitters and superposition coding at interfering transmitters leads to an inner bound within a quantifiable gap to the capacity region. For Gaussian MAC-IC-MAC, we show the Etkin-Tse-Wang type coding scheme achieves a one bit gap approximate capacity region, hence the GDoF region is fully characterized. The symmetric GDoF curve and example discussed reveal the role of non-interfering transmitters in improving the cell edge spectrum efficiency. The insights obtained here may also be relevant to fully-connected K -transmitter, 2-receiver interference networks when the interference from all but one transmitter in each cell are sufficiently weak to be at the noise level (and they are treated as noise).

APPENDIX A
PROOF OF THEOREM 6

We prove the achievability through a random coding argument. We fix some $p(q, u_{a0}, x_a, u_{b0}, x_b)$, and obtain reliability conditions in the form of partial sum-rate restrictions, which gives an achievable region with respect to a specific joint distribution, with the overall inner region being the convex hull of the union over all choices of joint distributions. With a little abuse of the notation, we use B_{i0} to denote the rate of the public messages from transmitter Tx i 0 within this section.

A. An Achievable Coding Scheme

- 1) Generate time sharing sequence q^n according to $p(q^n) = \prod_{t=1}^n p(q_t)$;
- 2) Tx i j , $j \neq 0$, generates $2^{nR_{ij}}$ sequences x_{ij}^n according to $p(x_{ij}^n|q^n) = \prod_{t=1}^n p(x_{ij,t}|q_t)$ and indexes them by $l_{ij} \in \{1, \dots, 2^{nR_{ij}}\}$;
- 3) Tx i 0 generates $2^{nB_{i0}}$ sequences u_{i0}^n according to $p(u_{i0}^n|q^n) = \prod_{k=1}^n p(u_{i0,k}|q_k)$ and indexes them by $h_{i0} \in \{1, \dots, 2^{nB_{i0}}\}$. For each $u_{i0}^n(h_{i0})$, generate $2^{n(R_{i0}-B_{i0})}$ sequences x_{i0}^n according to $p(x_{i0}^n|u_{i0}^n(h_{i0}), q^n) = \prod_{k=1}^n p(x_{i0,k}|u_{i0,k}(h_{i0}), q_k)$ and index them by $(h_{i0}, l_{i0}) \in \{1, \dots, 2^{nB_{i0}}\} \times \{1, \dots, 2^{n(R_{i0}-B_{i0})}\}$;
- 4) Once the three codebooks are generated, they are fixed over time and revealed to receivers Rx a and Rx b ;
- 5) A K_i -tuple message $(m_{i0}, \dots, m_{i(K_i-1)}) = ((h_{i0}, l_{i0}), l_{i1}, \dots, l_{i(K_i-1)})$ is encoded to $(x_{i0}^n(h_{i0}, l_{i0}), x_{i1}^n(l_{i1}), \dots, x_{i(K_i-1)}^n(l_{i(K_i-1)}))$ at Tx i 0-Tx $i(K_i-1)$ and sent to the channel;
- 6) Upon receiving y_i^n , Rx i declares its decoded messages $(\hat{m}_{i0}, \dots, \hat{m}_{i(K_i-1)})$ as the unique index pair $((\hat{h}_{i0}, \hat{l}_{i0}), \hat{l}_{i1}, \dots, \hat{l}_{i(K_i-1)})$ for which $q^n, x_{i0}^n(\hat{h}_{i0}, \hat{l}_{i0}), x_{i1}^n(\hat{l}_{i1}), \dots, x_{i(K_i-1)}^n(\hat{l}_{i(K_i-1)}), u_{i0}^n(\hat{h}_{i0})$ and y_i^n are jointly typical, for some $\hat{h}_{i'0} \in \{1, \dots, 2^{nB_{i'0}}\}$. If such an index pair cannot be found, the decoder declares an error;
- 7) An error occurs if $(\hat{M}_{i0}, \dots, \hat{M}_{i(K_i-1)}) \neq (M_{i0}, \dots, M_{i(K_i-1)})$.

For arbitrary Ω_i and Υ_i , any possible error events at Rx i falls into one of the four classes listed below:

- $\hat{h}_{i0} = 1, \hat{l}_{i0} \neq 1, \hat{l}_{\Upsilon_i \setminus \{i0\}} \neq \mathbf{1}, \hat{l}_{i'0} = 1$;
- $\hat{l}_{\Omega_i} \neq \mathbf{1}, \hat{l}_{i'0} = 1$;
- $\hat{h}_{i0} = 1, \hat{l}_{i0} \neq 1, \hat{l}_{\Upsilon_i \setminus \{i0\}} \neq \mathbf{1}, \hat{l}_{i'0} \neq 1$;
- $\hat{l}_{\Omega_i} \neq \mathbf{1}, \hat{l}_{i'0} \neq 1$.

which can be bounded using set functions A, B, E and G as following, plus the non-negativity of B_{a0} and $R_{a0} - B_{a0}$,

$$R_{\Upsilon_a} - B_{a0} \leq I(X_{\Upsilon_a}; Y_a | X_{\bar{\Upsilon}_a}, U_{a0}, U_{b0}, Q) = A_{\Upsilon_a} \quad (40)$$

$$R_{\Omega_a} \leq I(X_{\Omega_a}; Y_a | X_{\bar{\Omega}_a}, U_{b0}, Q) = B_{\Omega_a} \quad (41)$$

$$R_{\Upsilon_a} - B_{a0} + B_{b0} \leq I(X_{\Upsilon_a}, U_{b0}; Y_a | X_{\bar{\Upsilon}_a}, U_{a0}, Q) = E_{\Upsilon_a} \quad (42)$$

$$R_{\Omega_a} + B_{b0} \leq I(X_{\Omega_a}, U_{b0}; Y_a | X_{\bar{\Omega}_a}, Q) = G_{\Omega_a} \quad (43)$$

$$-B_{a0} \leq 0 = C_{\Omega_a} \quad (44)$$

$$B_{a0} - R_{\Upsilon_a} \leq 0 = D_{\Upsilon_a} \quad (45)$$

$$R_{\mathcal{R}_b} - B_{b0} \leq I(X_{\mathcal{R}_b}; Y_b | X_{\bar{\mathcal{R}}_b}, U_{b0}, U_{a0}, Q) = A_{\mathcal{R}_b} \quad (46)$$

$$R_{\Omega_b} \leq I(X_{\Omega_b}; Y_b | X_{\bar{\Omega}_b}, U_{a0}, Q) = B_{\Omega_b} \quad (47)$$

$$R_{\mathcal{R}_b} - B_{b0} + B_{a0} \leq I(X_{\mathcal{R}_b}, U_{a0}; Y_b | X_{\bar{\mathcal{R}}_b}, U_{b0}, Q) = E_{\mathcal{R}_b} \quad (48)$$

$$R_{\Omega_b} + B_{a0} \leq I(X_{\Omega_b}, U_{a0}; Y_b | X_{\bar{\Omega}_b}, Q) = G_{\Omega_b} \quad (49)$$

$$-B_{b0} \leq 0 = C_{\Omega_b} \quad (50)$$

$$B_{b0} - R_{\mathcal{R}_b} \leq 0 = D_{\mathcal{R}_b} \quad (51)$$

B. Fourier Motzkin Elimination

The elegant structure of the initial inequality system given above allows us to apply Fourier Motzkin elimination analytically. Without loss of generality, we first eliminate B_{a0} . Note all the lower bounds in the system are contributed by $A_{\mathcal{R}_a}$, $E_{\mathcal{R}_a}$ and C_{Ω_a} , and upper bounds by D_{Ω_a} , $E_{\mathcal{R}_b}$ and G_{Ω_b} , therefore B_{a0} is eliminated when we exhaustively sum $A_{\mathcal{R}_a}$, $E_{\mathcal{R}_a}$ and C_{Ω_a} with D_{Ω_a} , $E_{\mathcal{R}_b}$ and G_{Ω_b} . The inequality system after eliminating B_{a0} becomes

$$\begin{aligned} 0 &\leq A_{\mathcal{R}_a} + D_{\mathcal{R}_a} * \\ R_{\mathcal{R}_a} + R_{\mathcal{R}_b} - B_{b0} &\leq A_{\mathcal{R}_a} + E_{\mathcal{R}_b} \\ R_{\mathcal{R}_a} + R_{\Omega_b} &\leq A_{\mathcal{R}_a} + G_{\Omega_b} \\ B_{b0} &\leq E_{\mathcal{R}_a} + D_{\mathcal{R}_a} \\ R_{\mathcal{R}_a} + R_{\mathcal{R}_b} &\leq E_{\mathcal{R}_a} + E_{\mathcal{R}_b} \\ R_{\mathcal{R}_a} + B_{b0} + R_{\Omega_b} &\leq E_{\mathcal{R}_a} + G_{\Omega_b} \\ -R_{\mathcal{R}_a} &\leq C_{\Omega_a} + D_{\mathcal{R}_a} * \\ R_{\mathcal{R}_b} - B_{b0} &\leq C_{\Omega_a} + E_{\mathcal{R}_b} * \\ R_{\Omega_b} &\leq C_{\Omega_a} + G_{\Omega_b} * \\ R_{\Omega_a} + B_{b0} &\leq G_{\Omega_a} \\ R_{\mathcal{R}_b} - B_{b0} &\leq A_{\mathcal{R}_b} \\ -B_{b0} &\leq C_{\Omega_b} \\ B_{b0} - R_{\mathcal{R}_b} &\leq D_{\mathcal{R}_b} \end{aligned}$$

the starred inequalities are clearly redundant when it is taken into consideration that, by the chain rule of mutual information, $A_{\mathcal{R}_i} \leq E_{\mathcal{R}_i}$ and $B_{\Omega_i} \leq G_{\Omega_i}$, . After removing these redundancies, we proceed to the elimination of B_{b0} . New lower bounds are provided by C_{Ω_b} , $A_{\mathcal{R}_b}$ and $A_{\mathcal{R}_a} + E_{\mathcal{R}_b}$, upper bounds are provided by D_{Ω_b} , $E_{\mathcal{R}_a}$, G_{Ω_b}

and $E_{\mathcal{R}_a} + G_{\Omega_b}$. Eliminating B_{b0} , a new system shows up as:

$$\begin{aligned}
0 &\leq C_{\Omega_b} + E_{\mathcal{R}_a} & * \\
R_{\mathcal{R}_b} &\leq A_{\mathcal{R}_b} + E_{\mathcal{R}_a} \\
R_{\mathcal{R}_a} + R_{\mathcal{R}_b} &\leq A_{\mathcal{R}_a} + E_{\mathcal{R}_b} + E_{\mathcal{R}_a} & * \\
-R_{\mathcal{R}_b} &\leq C_{\Omega_b} + D_{\mathcal{R}_b} & * \\
0 &\leq A_{\mathcal{R}_b} + D_{\mathcal{R}_b} & * \\
R_{\mathcal{R}_a} &\leq A_{\mathcal{R}_a} + E_{\mathcal{R}_b} + D_{\mathcal{R}_b} \\
R_{\Omega_a} &\leq C_{\Omega_b} + G_{\Omega_a} & * \\
R_{\mathcal{R}_b} + R_{\Omega_a} &\leq A_{\mathcal{R}_b} + G_{\Omega_a} \\
R_{\mathcal{R}_a} + R_{\mathcal{R}_b} + R_{\Omega_a} &\leq A_{\mathcal{R}_a} + E_{\mathcal{R}_b} + G_{\Omega_a} \\
R_{\mathcal{R}_a} + R_{\Omega_b} &\leq C_{\Omega_b} + E_{\mathcal{R}_a} + G_{\Omega_b} & * \\
R_{\mathcal{R}_b} + R_{\mathcal{R}_a} + R_{\Omega_b} &\leq A_{\mathcal{R}_b} + E_{\mathcal{R}_a} + G_{\Omega_b} \\
R_{\mathcal{R}_a} + R_{\mathcal{R}_b} + R_{\mathcal{R}_a} + R_{\Omega_b} &\leq A_{\mathcal{R}_a} + E_{\mathcal{R}_b} + E_{\mathcal{R}_a} + G_{\Omega_b} & * \\
R_{\mathcal{R}_a} + R_{\Omega_b} &\leq A_{\mathcal{R}_a} + G_{\Omega_b} \\
R_{\mathcal{R}_a} + R_{\mathcal{R}_b} &\leq E_{\mathcal{R}_a} + E_{\mathcal{R}_b}
\end{aligned}$$

The starred inequalities can be identified as redundancies easily. Again, similar to the previous step, we obtain a region given as

$$R_{\Omega_a} \leq B_{\Omega_a} \tag{52}$$

$$R_{\mathcal{R}_a} \leq A_{\mathcal{R}_a} + E_{\mathcal{R}_b} \tag{53}$$

$$R_{\Omega_b} \leq B_{\Omega_b} \tag{54}$$

$$R_{\mathcal{R}_b} \leq A_{\mathcal{R}_b} + E_{\mathcal{R}_a} \tag{55}$$

$$R_{\mathcal{R}_a} + R_{\Omega_b} \leq A_{\mathcal{R}_a} + G_{\Omega_b} \tag{56}$$

$$R_{\mathcal{R}_b} + R_{\Omega_a} \leq A_{\mathcal{R}_b} + G_{\Omega_a} \tag{57}$$

$$R_{\mathcal{R}_a} + R_{\mathcal{R}_b} \leq E_{\mathcal{R}_a} + E_{\mathcal{R}_b} \tag{58}$$

$$R_{\mathcal{R}_a} + R_{\mathcal{R}_b} + R_{\Omega_a} \leq A_{\mathcal{R}_a} + E_{\mathcal{R}_b} + G_{\Omega_a} \tag{59}$$

$$R_{\mathcal{R}_b} + R_{\mathcal{R}_a} + R_{\Omega_b} \leq A_{\mathcal{R}_b} + E_{\mathcal{R}_a} + G_{\Omega_b} \tag{60}$$

It remains to show that inequalities (53) and (55) are redundant. Due to the symmetry of the channel, it is sufficient to show (53) is redundant.

Denote the region in Theorem 6 as \mathcal{R}_0 for brevity, the region by inequalities (52)-(60) as \mathcal{R}_1 , $\mathcal{R}_2 = \mathcal{R}_0 \setminus \mathcal{R}_1$ and $\mathcal{R}_3 = \{(R_a, R_b) \in \mathcal{R}_1 : U_{a0} = \emptyset\}$. Since $\mathcal{R}_3 \subseteq \mathcal{R}_1$, it is clear that

$$\mathcal{R}_2 \cap \mathcal{R}_3 = \emptyset \quad (61)$$

Region \mathcal{R}_3 can be obtained by setting $U_{a0} = \emptyset$ in initial system (40)-(51), and eliminating B_{b0} , which is

$$R_{\Omega_a} \leq I(X_{\Omega_a}; Y_a | X_{\bar{\Omega}_a}, U_{b0}, Q) \quad (62)$$

$$R_{\Omega_b} \leq I(X_{\Omega_b}; Y_b | X_{\bar{\Omega}_b}, Q) \quad (63)$$

$$\begin{aligned} R_{\Omega_a} + R_{\mathcal{R}_b} &\leq I(X_{\Omega_a}, U_{b0}; Y_a | X_{\bar{\Omega}_a}, Q) \\ &\quad + I(X_{\mathcal{R}_b}; Y_b | X_{\bar{\mathcal{R}}_b}, U_{b0}, Q) \end{aligned} \quad (64)$$

Region \mathcal{R}_2 is constituted by all the inequalities for region \mathcal{R}_0 and $R_{\mathcal{R}_a} \geq A_{\mathcal{R}_a} + E_{\mathcal{R}_b}$, here we explicitly write them as below

$$R_{\Omega_a} \leq B_{\Omega_a} \quad (65)$$

$$-R_{\mathcal{R}_a} \leq -A_{\mathcal{R}_a} - E_{\mathcal{R}_b} \quad (66)$$

$$R_{\Omega_b} \leq B_{\Omega_b} \quad (67)$$

$$R_{\mathcal{R}_a} + R_{\Omega_b} \leq A_{\mathcal{R}_a} + G_{\Omega_b} \quad (68)$$

$$R_{\mathcal{R}_b} + R_{\Omega_a} \leq A_{\mathcal{R}_b} + G_{\Omega_a} \quad (69)$$

$$R_{\mathcal{R}_a} + R_{\mathcal{R}_b} \leq E_{\mathcal{R}_a} + E_{\mathcal{R}_b} \quad (70)$$

$$R_{\mathcal{R}_a} + R_{\mathcal{R}_b} + R_{\Omega_a} \leq A_{\mathcal{R}_a} + E_{\mathcal{R}_b} + G_{\Omega_a} \quad (71)$$

$$R_{\mathcal{R}_b} + R_{\mathcal{R}_a} + R_{\Omega_b} \leq A_{\mathcal{R}_b} + E_{\mathcal{R}_a} + G_{\Omega_b} \quad (72)$$

From inequality (65), we know

$$R_{\Omega_a} \leq B_{\Omega_a} = I(X_{\Omega_a}; Y_a | X_{\bar{\Omega}_a}, U_{b0}, Q) \quad (73)$$

adding inequalities (66) and (68), we have

$$\begin{aligned} R_{\Omega_b} &\leq G_{\Omega_b} - E_{\mathcal{R}_b} \\ &= I(X_{\Omega_b}, U_{a0}; Y_b | X_{\bar{\Omega}_b}, Q) - I(X_{\mathcal{R}_b}, U_{a0}; Y_b | X_{\bar{\mathcal{R}}_b}, U_{b0}, Q) \\ &= H(Y_b | X_{\bar{\Omega}_b}, Q) - H(Y_b | X_{\bar{\mathcal{R}}_b}, U_{b0}, Q) \\ &\leq H(Y_b | X_{\bar{\Omega}_b}, Q) - H(Y_b | X_{\bar{\mathcal{R}}_b}, X_{\mathcal{R}_b}, Q) \\ &= H(Y_b | X_{\bar{\Omega}_b}, Q) - H(Y_b | X_{\Omega_b}, X_{\bar{\Omega}_b}, Q) \\ &= I(X_{\Omega_b}; Y_b | X_{\Omega_b}, Q) \end{aligned} \quad (74)$$

adding inequalities (66) and (71), we have

$$\begin{aligned}
R_{\mathcal{R}_b} + R_{\Omega_a} &\leq G_{\Omega_a} \\
&= I(X_{\Omega_a}, U_{b0}; Y_a | X_{\bar{\Omega}_a}, Q) \\
&\leq I(X_{\Omega_a}, U_{b0}; Y_a | X_{\bar{\Omega}_a}, Q) \\
&\quad + I(X_{\mathcal{R}_b}; Y_b | X_{\bar{\mathcal{R}}_b}, U_{b0}, Q)
\end{aligned} \tag{75}$$

Note inequalities (62)-(64) are identical to (73)-(75), which means

$$\mathcal{R}_2 \subseteq \mathcal{R}_3 \tag{76}$$

it then contradicts (61). Hence, (53) and (55) are both redundancies, which completes the proof.

APPENDIX B

PROOF OF THEOREM 9

For some fixed $p(q)p(t_a, x_a|q)p(t_b, x_b|q)$, applying Fano's inequality, chain rule, independence and Markov chain property, the sum rate R_{Ω_a} on any subset Ω_a can be upper bounded by set functions B_{Ω_a} and G_{Ω_a} . To upper bound $R_{\mathcal{R}_a}$, we give genie information T_a^n (as shown in Fig. 8) to help decoding at Rx a . Given that adding side information will not decrease the rate, as well as T_a^n is independent of S_a^n conditioning on X_{a0}^n , $R_{\mathcal{R}_a}$ can be upper bounded by $A_{\mathcal{R}_a}$ and $E_{\mathcal{R}_a}$.

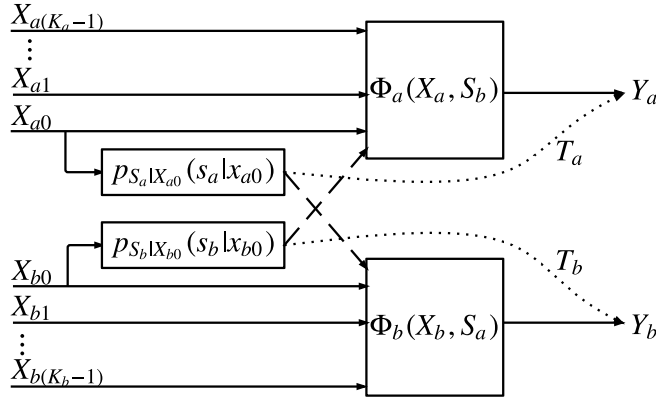


Fig. 8. Genie aided semi-deterministic MAC-IC-MAC, where a genie provides information T_{a0} to Rx $a0$ and T_{b0} to Rx $b0$.

$$\begin{aligned}
nR_{\mathcal{R}_a} &\leq I(X_{\mathcal{R}_a}^n; Y_a^n, X_{\mathcal{R}_a}^n, T_a^n, X_{b0}^n) + n\epsilon_n \\
&= I(X_{\mathcal{R}_a}^n; T_a^n | X_{\mathcal{R}_a}^n, X_{b0}^n) \\
&\quad + I(X_{\mathcal{R}_a}^n; Y_a^n | X_{\mathcal{R}_a}^n, T_a^n, X_{b0}^n) + n\epsilon_n \\
&= I(X_{\mathcal{R}_a}^n; T_a^n) + I(X_{\mathcal{R}_a}^n; Y_a^n | X_{\mathcal{R}_a}^n, T_a^n, X_{b0}^n) + n\epsilon_n \\
&= H(T_a^n) - H(T_a^n | X_{\mathcal{R}_a}^n) + H(Y_a^n | X_{\mathcal{R}_a}^n, T_a^n, X_{b0}^n)
\end{aligned}$$

$$\begin{aligned}
& -H(Y_a^n | X_{\mathcal{R}_a}^n, X_{\mathcal{T}_a}^n, T_a^n, X_{b0}^n) + n\epsilon_n \\
& = H(T_a^n) - H(T_a^n | X_{\mathcal{R}_a}^n) + H(Y_a^n | X_{\mathcal{R}_a}^n, T_a^n, X_{b0}^n) \\
& \quad - H(S_b^n | X_{b0}^n) + n\epsilon_n \\
& \leq H(T_a^n) + \sum_{t=1}^n [H(Y_{at} | X_{\mathcal{R}_{at}}, T_{at}, X_{b0,t}) \\
& \quad - H(T_{at} | X_{\mathcal{R}_{at}}) - H(S_{bt} | X_{b0,t})] + n\epsilon_n \\
& = \bar{A}_{\mathcal{R}_a} - H(S_b | X_{b0}) + H(T_a^n) + n\epsilon_n
\end{aligned} \tag{77}$$

$$\begin{aligned}
nR_{\Omega_a} & \leq I(X_{\Omega_a}^n; Y_a^n) + n\epsilon_n \\
& = I(X_{\Omega_a}^n; Y_a^n | X_{\bar{\Omega}_a}^n, X_{b0}^n) + n\epsilon_n \\
& = H(Y_a^n | X_{\bar{\Omega}_a}^n, X_{b0}^n) - H(Y_i^n | X_{\Omega_a}^n, X_{\bar{\Omega}_a}^n, X_{b0}^n) + n\epsilon_n \\
& = H(Y_a^n | X_{\bar{\Omega}_a}^n, X_{b0}^n) - H(S_b^n | X_{b0}^n) + n\epsilon_n \\
& \leq \sum_{t=1}^n [H(Y_{at} | X_{\bar{\Omega}_{at}}, X_{b0,t}) - H(S_{bt} | X_{b0,t})] + n\epsilon_n \\
& = \bar{B}_{\Omega_a} + n\epsilon_n
\end{aligned} \tag{78}$$

$$\begin{aligned}
nR_{\mathcal{R}_a} & \leq I(X_{\mathcal{R}_a}^n; Y_a^n, X_{\mathcal{T}_a}^n, T_a^n) + n\epsilon_n \\
& = I(X_{\mathcal{R}_a}^n; T_a^n | X_{\mathcal{T}_a}^n) + I(X_{\mathcal{R}_a}^n; Y_a^n | X_{\mathcal{T}_a}^n, T_a^n) + n\epsilon_n \\
& = I(X_{\mathcal{R}_a}^n; T_a^n) + I(X_{\mathcal{R}_a}^n; Y_a^n | X_{\mathcal{T}_a}^n, T_a^n) + n\epsilon_n \\
& = H(T_a^n) - H(T_a^n | X_{\mathcal{R}_a}^n) + H(Y_a^n | X_{\mathcal{R}_a}^n, T_a^n) \\
& \quad - H(Y_a^n | X_{\mathcal{R}_a}^n, X_{\mathcal{T}_a}^n, T_a^n) + n\epsilon_n \\
& = H(T_a^n) - H(T_a^n | X_{\mathcal{R}_a}^n) + H(Y_a^n | X_{\mathcal{T}_a}^n, T_a^n) \\
& \quad - H(S_b^n) + n\epsilon_n \\
& \leq \sum_{t=1}^n [H(Y_{at} | X_{\mathcal{T}_{at}}, T_{at}) - H(T_{at} | X_{\mathcal{R}_{at}})] \\
& \quad + H(T_a^n) - H(S_b^n) + n\epsilon_n \\
& = \bar{E}_{\mathcal{R}_a} + H(T_a^n) + n\epsilon_n
\end{aligned} \tag{79}$$

$$\begin{aligned}
nR_{\Omega_a} & \leq I(X_{\Omega_a}^n; Y_a^n) + n\epsilon_n \\
& = I(X_{\Omega_a}^n; Y_a^n | X_{\bar{\Omega}_a}^n) + n\epsilon_n \\
& = H(Y_a^n | X_{\bar{\Omega}_a}^n) - H(Y_i^n | X_{\Omega_a}^n, X_{\bar{\Omega}_a}^n) + n\epsilon_n \\
& = H(Y_a^n | X_{\bar{\Omega}_a}^n) - H(S_b^n) + n\epsilon_n
\end{aligned}$$

$$\begin{aligned}
&\leq \sum_{t=1}^n H(Y_{at}|X_{\bar{\Omega}_a t}) - H(S_b^n) + n\epsilon_n \\
&= \bar{G}_{\Omega_a} + H(S_b|X_{b0}) - H(S_b^n) + n\epsilon_n
\end{aligned} \tag{80}$$

Similarly, we could have 4 upper bounds on rate R_b too,

$$nR_{\mathcal{R}_b} \leq \bar{A}_{\mathcal{R}_b} - H(S_a|X_{a0}) + H(T_b^n) + n\epsilon_n \tag{81}$$

$$nR_{\Omega_b} \leq \bar{B}_{\Omega_b} + n\epsilon_n \tag{82}$$

$$nR_{\mathcal{R}_b} \leq \bar{E}_{\mathcal{R}_b} + H(T_b^n) + n\epsilon_n \tag{83}$$

$$nR_{\Omega_b} \leq \bar{G}_{\Omega_b} + H(S_a|X_{a0}) - H(S_a^n) + n\epsilon_n \tag{84}$$

The seven inequalities in Theorem 9 can be obtained by these linear combination of inequalities above: (78), (82), (77)+(80), (81)+(84), (79)+(83), (77)+(83)+(80) and (81)+(79)+(84).

APPENDIX C

PROOF OF THEOREM 10

For a semi-deterministic MAC-IC-MAC, the terms in Theorem 6 satisfy

$$A_{\mathcal{R}_i} \geq H(Y_i|X_{\mathcal{R}_i}, U_{i0}, X_{i'0}, Q) - H(S_{i'}|U_{i'}, Q) = A'_{\mathcal{R}_i}$$

$$B_{\Omega_i} \geq H(Y_1|X_{1b}, X_2, Q) - H(S_2|U_2, Q) = B'_{\Omega_i}$$

$$E_{\mathcal{R}_i} = H(Y_1|U_{1b}, Q) - H(S_2|U_2, Q) = E'_{\mathcal{R}_i}$$

$$G_{\Omega_i} = H(Y_1|Q) - H(S_2|U_2, Q) = G'_{\Omega_i}$$

because replacing U_{i0} with X_{i0} in the positive terms in Definition 6 would only reduce the corresponding conditional entropy. For some $(Q, U_{a0}, X_a, U_{b0}, X_b)$ with joint distribution $p(q)p(u_{a0}, x_a|q)p(u_{b0}, x_b|q)$, define $\mathcal{R}'_{\text{in}}(Q, U_{a0}, X_a, U_{b0}, X_b)$ to be $\mathcal{R}(R_a, R_b, A', B', E', G')$.

Next, we put a further restriction on U_{i0} in $\mathcal{R}'_{\text{in}}(Q, U_{a0}, X_a, U_{b0}, X_b)$ such that

$$p_{U_{i0}|X_{i0}, Q}(u_{i0}|x_{i0}, q) = p_{S_i|X_{i0}}(u_{i0}|x_{i0}) = p_{T_i|X_{i0}}(u_{i0}|x_{i0})$$

which results in another region $\mathcal{R}''_{\text{in}}(Q, T_a, X_a, T_b, X_b)$, so that it can be concluded that

$$\begin{aligned}
\mathcal{R}''_{\text{in}} &\triangleq \bigcup_{\substack{Q, T_a, X_a \\ T_b, X_b}} \mathcal{R}''_{\text{in}}(Q, T_a, X_a, T_b, X_b) \\
&\subseteq \mathcal{R}_{\text{in}}
\end{aligned}$$

On the other hand, we have

$$\begin{aligned}
I(X_i; S_i|T_i, Q) &= H(S_i|T_i, Q) - H(S_i|X_i, T_i, Q) \\
&= H(S_i|T_i, Q) - H(S_i|X_i, Q)
\end{aligned}$$

Hence given any rate tuple $(R_a, R_b) \in \mathcal{R}_{\text{out}}$, it is clear that $(R_a - I(X_b; S_b|T_b, Q)\mathbf{1}_{1 \times K_a}, R_b - I(X_a; S_a|T_a, Q)\mathbf{1}_{1 \times K_b}) \in \mathcal{R}''_{\text{in}} \subseteq \mathcal{R}_{\text{in}}$, which proves the quantifiable gap.

APPENDIX D
PROOF OF THEOREM 14

According to Theorem 9, we characterize the outer bound on the capacity region of MAC-IC-MAC by computing the set functions in Definition 9. We choose the genie random variables $T_i = h_{i0,i'} X_{i0} + Z'_{i'}$, where $Z'_{i'} \sim \mathcal{CN}(0, 1)$ and independent of $Z_{i'}$ and X_i . For conciseness, we compute $\bar{A}_{\mathcal{R}_i}$ for instance,

$$\begin{aligned}
\bar{A}_{\mathcal{R}_i} &= h(Y_i | X_{\mathcal{R}_i}, T_i, X_{i'0}, Q) - h(S_{i'} | X_{i'}, Q) \\
&= \frac{1}{n} \sum_{t=1}^n [h(Y_{it} | X_{\mathcal{R}_{it}}, T_{it}, X_{i'0t}) - h(S_{i't} | X_{i't})] + n\epsilon \\
&= \frac{1}{n} \sum_{t=1}^n \left[h\left(\sum_{ij \in \mathcal{R}_i} |h_{ij,i}|^2 X_{ij,t} + Z_{it}, |h_{i0,i'}|^2 X_{i0,t} + Z'_{i't}\right) \right. \\
&\quad \left. - h(|h_{i0,i}|^2 X_{i0,t} + Z'_{i't}) - h(Z_{i't}) \right] + n\epsilon \\
&\leq \frac{1}{n} \sum_{t=1}^n \log \left(1 + \sum_{ij \in \mathcal{R}_i \setminus \{i0\}} (|h_{ij,i}|^2 P_{ij,t}) \right. \\
&\quad \left. + \frac{|h_{i0,i}|^2 P_{i0,t}}{1 + |h_{i0,i'}|^2 P_{i0,t}} \right) + n\epsilon \\
&\leq \log \left(1 + \sum_{ij \in \mathcal{R}_i \setminus \{i0\}} \left(|h_{ij,i}|^2 \frac{1}{n} \sum_{t=1}^n P_{ij,t} \right) \right. \\
&\quad \left. + \frac{|h_{i0,i}|^2 \frac{1}{n} \sum_{t=1}^n P_{i0,t}}{1 + |h_{i0,i'}|^2 \frac{1}{n} \sum_{t=1}^n P_{i0,t}} \right) + n\epsilon \\
&\leq \log \left(1 + \sum_{ij \in \mathcal{R}_i \setminus \{i0\}} |h_{ij,i}|^2 P_{ij} + \frac{|h_{i0,i}|^2 P_{i0}}{1 + |h_{i0,i'}|^2 P_{i0}} \right) + n\epsilon \\
&= \log \left(1 + \sum_{ij \in \mathcal{R}_i \setminus \{i0\}} \left| \text{SNR}_{ij,i} + \frac{\text{SNR}_{i0,i}}{1 + \text{INR}_{i0,i'}} \right| \right) + n\epsilon
\end{aligned}$$

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